Cognitive Set Theory: A Summary

Alec Rogers

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ABSTRACT: This essay is an overview of Cognitive Set Theory. Cognitive Set Theory is a formal model of physical and mental spaces, the decomposition of those spaces into objects, and the relationships between those spaces and objects. Within mental space, an important distinction is made between perceptual (sensory, sub-symbolic) and conceptual (symbolic) spaces. The branches of mathematics known as mereology and set theory are used to respectively model those perceptual and conceptual spaces.

Introduction

Cognitive Set Theory is an attempt to formalize a mathematical grounding for cognition. It includes the scope of symbolic logic, and as its name implies, it uses set theory (and Boolean algebra) to do so. As modern psychology recognizes, however, rationality is only a small part of our overall cognitive capacity. Therefore, any mathematical account of mind must be of a broader scope than any symbolic description. Further, to model the sub-symbolic aspects of mind, set theory is not appropriate: therefore, mereology (the mathematics of parts and wholes) is used to model these perceptual (sub-symbolic) aspects of mind.

Cognitive Set Theory (CST) is a mathematical model, and it does not describe the neuroscience or implementation of minds. Despite the fact that it is formal model, however, it is not concerned with purely mathematical aims such as a reduction in the number of axioms: it is intended to mirror the major aspects of human cognition, however axiomatically complicated that may be. CST is also related to the field of numerical cognition, but unlike that field, its goal is not to map numbers on to various brain structures. Finally, although it is related to machine learning, it is a model of natural intelligence, not artificial intelligence, so it is less concerned with efficiency than ability to explain human behavior.

Related Work

Psychology: Dual Process Theory

Cognitive Set Theory is related to a psychological model called Dual Process Theory, which has been popularized by numerous authors such as Malcolm Gladwell and the Nobel prize winner Daniel Kahneman. According to Dual Process Theory, the mind is composed of a fast, parallel, highly emotional system (System 1) and a slower, serial, less emotional system (System 2). Cognitive Set Theory offers a particular interpretation of these Systems from a subjective point of view: System 1 is experienced perceptually (as percepts) and System 2 is experienced conceptually (as concepts or thoughts). For reference, the following properties are commonly associated with each system (Evans and Stanovich, 2013):

System 1: Independent of working memory, fast, parallel, associative, knows implicitly, relies on basic emotions.

System 2: Utilizes working memory, slow, serial, rule-based, knows explicitly, relies on complex emotions.

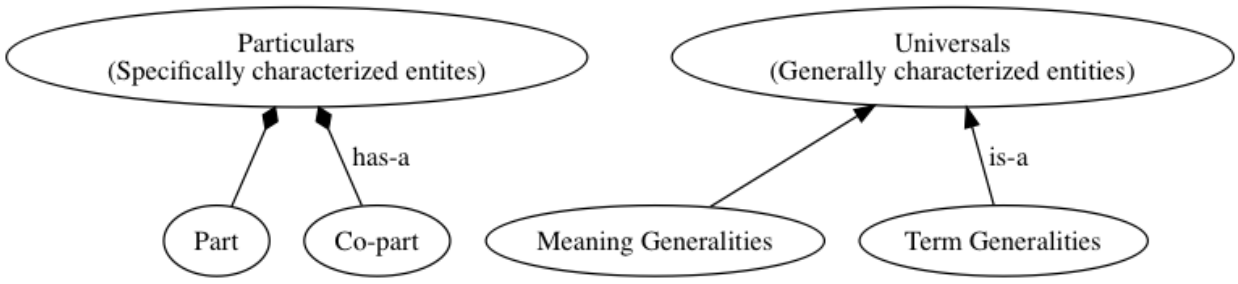
Linguistically, this distinction is similar (if not identical) to the distinction between semantics and syntax.

Philosophy: Particulars and Universals

The psychological/epistemological division of cognition into two systems is closely related to an ontological distinction in philosophy between what are called particulars and universals. For example, the horse named "Ed" is a particular thing, and the property (or collection of properties) known as

"horseness" is a universal thing. In contrast to our common understanding, the Greek philosopher Plato argued that universals (or Platonic forms) are real, and that particulars are unreal (like shadows cast on the wall of a cave: see Armstrong, 1989). In Tibetan Buddhism, the same distinction results in *specifically characterized entities* and *generally characterized entities*, each of which is cognized by a different type of mind. Generally characterized entities can be further divided, such that *term generalities* are composed of *meaning generalities* (Lati, 2008).

The following diagram illustrates the distinction between particulars and universals, how particulars compose using meronymic (has-a) relationships, and how universals compose using taxonomic (is-a) relationships.



Cognitive Set Theory

The Domain

In Cognitive Set Theory, both physical (ontological) reality and mental (epistemic) reality are structured as N-dimensional spaces. Epistemic space is further divided into perceptual (sensory) space and conceptual (abstract) space, a division that represents a categorical distinction because of the corresponding difference in dimensionality between those spaces. Objects within a given space are separated from one another by *nominal* boundaries: in other words, the boundaries themselves are not a part of the domain which they serve to divide. For example, a one-dimensional line can be divided by (imputed) conceptual points on that line, but those points are not assumed to *constitute* that line, since they are not of the same dimensionality.

It is essential to model the relationships between the physical and various mental spaces, which are related to the mind/body problem and the problem of qualia. A common psychological model of how these spaces interact is one in which the subjective (epistemic) space is a referential map of the objective (ontological) space. That subjective space may in turn be mapped by successive conceptual spaces. In order to correctly navigate within the world, these maps must be accurate: the concept "bicycle" should refer to the subjective percept 'bicycle', which should in turn refer to the object in the world known as a bicycle. Further, each of these things must exist within a similar isomorphic context within its own domain: just as the perception of a bike contains two perceptions of wheels, the concept of a bike contains two wheel-concepts.

A primary focus of CST are the mathematical formalisms that best describe our perceptual and conceptual processes: therefore, these different formalisms are discussed in the next two sections.

Perceptual Mathematics

Because space is used as a metaphor for mind, the study of shapes (topology) is naturally correlated with the study of percepts (or perceptual objects). Most modern topology uses set theory as its mathematical foundation, with points as the elements of those sets, but defining shapes *in terms of* sets of points is not psychologically valid. In particular, although we may divide our consciousness into many small parts, there is no reason to assume that our psychological reality *begins* with points, and a number of reasons not to do so.

The elements of the CST spatial model are not points: they are (non-infinitesimal) percepts formed by the division of a larger space or whole. The replacement of points with parts in this context, which relies on developmental psychology, coincides with the rejection of point-set topology in favor of mereotopology[1]. According to developmental psychology, the first parts (or internal representations) are created before six months of age, which corresponds to the developmental stage called *object permanence* (Piaget, 1963). The bodily sense of self is a strong candidate for the first permanent object from a biological point of view, because bodily inputs change much less than external inputs (e.g. the visual world changes when we turn our head, but the (proprioceptive) feeling inside our head simultaneously changes relatively little[2]). Regardless of the first perceptual division, the first perceptual part may be further divided, creating what is called a mereonomy (or part-hierarchy).

The important observation is that this mereotopological process is holistic and top-down, as opposed to the reductionistic and bottom-up process of point-set topology. Even though it may be possible to describe the mind from a purely

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[1] The axioms of mereotopology are explored further in works by Achille Varzi (Varzi 1999) and Peter Simmons (Simons 1987).

[2] The fact that the neuronal constancy of inputs would determine categorization is implied by the neural dictum: "features that fire together, wire together".

bottom-up (point-set topological) perspective[3], doing so creates several counter-intuitive notions such as infinity, infinitesimal quantities, and the difference between open and closed surfaces. In fact, these concerns motivated a movement within mathematics known as intuitionism, which attempts to ground mathematics in an intuitively-compatible way.

Conceptual Mathematics

In Cognitive Set Theory, set theory forms the mathematical basis for conceptuality. This choice is not surprising, since sets (or *classes*, as sets were once known) were historically defined as collections of objects, a definition that leads naturally to the idea of generalization. The elements of these (first-order) sets are the parts of the world that result from mereological analysis, which are collected into a set (or whole[4]) and usually designated with a symbol. These first-order sets (or concepts) may be subsequently collected into higher-order sets (or concepts). This iterative and extensional construction of sets is presumably recursive, and enabled by short-term memory. As opposed to parts, the ability of sets to isolate their constituents from their (spatiotemporal) context leads to taxonomies (or abstract hierarchies).

The correspondence between mathematical sets and psychological concepts has a number of important features:

1) Sets are single entities, although sets may contain (and in turn compose) a plurality of things.

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[3] This is analogous to the fact that we may (conceptually) reason about our intuitive mind, yet that does not imply that our intuitive mind is itself grounded in rational thought.

[4] This collection may be done by enumeration (definition by extension), or it may be done in terms of a formula (definition by intension).

2) Sets are typically defined with respect to their contents, independently of other sets.

3) Sets are in some sense opaque; for example, set membership is not transitive, unlike part membership[5].

4) The depth of the set hierarchy entailed during the composition of a concept or generalization corresponds to its psychological level of abstraction, or its dimensionality.

[5] Because sets are opaque, they may be used to constitute a single space in different ways.

Conclusion

Cognitive Set Theory is a mental model that uses space as an analogy for the mind, mereology to describe percepts, and set theory to describe concepts. Although this model is in many ways not novel, its combination of formal mathematics and psychology allows us to talk about and understand minds in a fairly precise way, which gives it the potential to be a *lingua franca* between multiple disciplines.

It should be clear that this model is not *complete*. Perhaps its biggest omission is that it does not model emotions, or explain *why* we form particular percepts and concepts of the world. But even without modeling affect, I believe it offers great promise for creating models of cognitive health, especially with respect to the balance between perception and conceptualization, or feeling and thinking.

Finally, the material presented here is a summary of an earlier and longer work. Although many of the key insights were presented here in a terse and clear manner, the interested reader is encouraged to have a look at the original book (Cognitive Set Theory) and several related essays at: http://cognitivesettheory.com .

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