The background of the slide is a photograph of a river winding through a lush, green forest. The river is in the foreground, flowing towards the background where it disappears into the trees. The forest is dense with various types of trees, including tall evergreens and deciduous trees with green foliage. The sky is visible in the upper part of the image, showing a clear blue color. The title text is overlaid on the upper half of the image.

# COGNITIVE SET THEORY:

*A Summary*

*Alec Rogers*

*2018 / 01 / 22*

# PERSONAL BACKGROUND

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- My name is Alec Rogers, I went to Reed college in the 90's, and graduated from PSU in the 00's.
- Cognitive Set Theory is my ongoing attempt to use set theory and mathematics as a model of human cognition.
- Much of my interest in cognition has to do with the cultivation of awareness and meditation.

# A SUMMARY

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*A set is a many which allows itself to be thought of as a one*

~ Georg Cantor

*This Knowledge that has no center or circumference, no inside or outside, ... [it] is a vast expanse of space* ~ Longchen

Rabjam

- Cognitive Set Theory is a formal model of physical and mental spaces.
- Within mental space, an important distinction is made between perceptual (sensory, sub-symbolic) and conceptual (symbolic) spaces.
- The branches of mathematics known as mereology and set theory are used to respectively model those perceptual and conceptual spaces.



# MOTIVATION

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- We need a high-level formal language for cognition.
- Cognitive Set Theory (CST) is a mathematical model, and it does not describe the neuroscience or implementation of minds.
- It is not concerned with purely mathematical aims, such as a reduction in the number of axioms: it is intended to mirror the major aspects of human cognition.
- It is a model of natural intelligence, not artificial intelligence, so it is concerned with explaining human behavior, not efficiency.



# MORE MOTIVATION

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- Cognitive Set Theory is an attempt to formalize a mathematical grounding for cognition.
- It includes the scope of symbolic logic, and uses set theory to do so.
- As modern psychology recognizes, however, rationality is only a small part of our overall cognitive capacity: we need a sub-symbolic logic.
- To model the sub-symbolic (perceptual) aspects of mind, we use mereology (the mathematics of parts and wholes).

# ONTOLOGY AND EPISTEMOLOGY : REAL THINGS

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- The domain of “real” (physical) things is spacetime.
- The philosophical question of which things are “truly real” corresponds to the investigation of *natural kinds* (in Western philosophy), or *pramana* (in Indian philosophy).
- The answer from both of these inquiries often takes a reductionistic approach: atoms (or quarks, or muons, or god particles) are real, or the five elements are real (air, water, fire, earth, space).
- According to that logic, larger entities are constructed out of those smaller entities (which are in some sense *more real*).

# ONTOLOGY AND EPISTEMOLOGY : IDEAL THINGS

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- As we move from ontology to epistemology, our question is analogous: what are *real* mental objects?
- If we adopt the convention that concepts (mental objects) reference physical objects, we may similarly posit a mental space that reflects physical space.
- Indian (abhidhamma) psychology maintains that the 52 mental factors are the mental atoms from which all aspects of mental life are composed.
- In Western psychology, concepts are studied primarily indirectly: the building blocks and artifacts of thought are studied, presumably due to an emphasis on the 3<sup>rd</sup>-person (scientific) perspective.



# PHILOSOPHY : PARTICULARS AND UNIVERSALS

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- Within mental space, the philosophical distinction between particulars and universals creates an important distinction:

**Particular:** a particular horse in a field at time  $t$ .

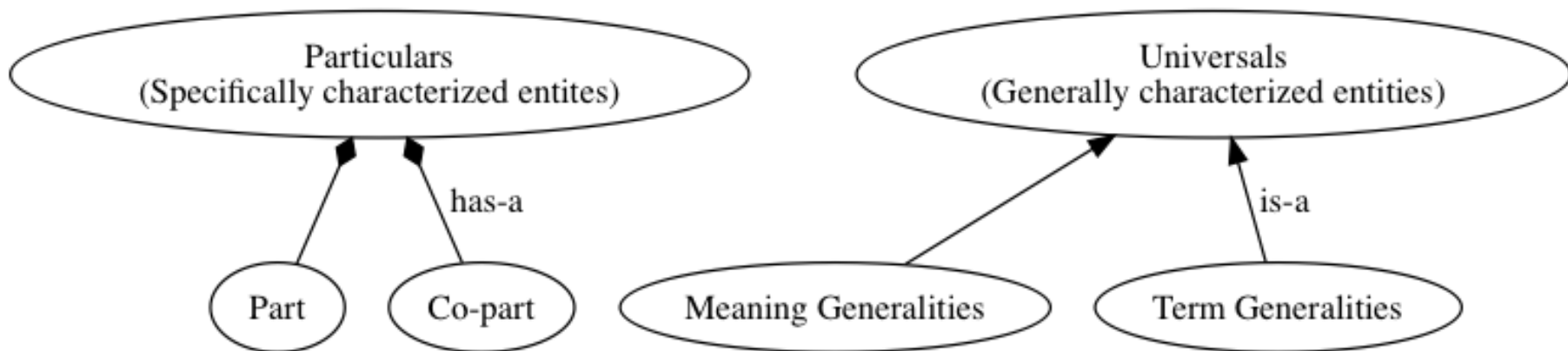
**Universal:** the property (or collection of properties) known as *horseness*.

- The same distinction exists in Buddhism (i.e. between *specifically characterized entities* and *generally characterized entities*). Generally characterized entities are further divided, resulting in *term generalities* and *meaning generalities*.

# MERONOMIES AND TAXONOMIES

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- The following diagram illustrates the distinction between particulars and universals, meronomies and taxonomies.



# MATHEMATICS : SET THEORY AND TOPOLOGY

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- Set theory was initially developed by Georg Cantor and Richard Dedekind in the 1870s, and developed by Russel, Whitehead, Zermelo and Fraenkel.
- Set theory is widely used as a framework to ground the rest of mathematics.
- Several axioms from set theory and point-set topology:

**Element:**  $\emptyset = \{ \}$

**Set:**  $\{ \emptyset \}$

**Composition:**  $\{ \emptyset, \{ \emptyset \} \}$

**Point:**  $\cdot$



# MATHEMATICS : MEREOLOGY

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- Mereology literally means the study of parts. It was developed by Stanisław Leśniewski in 1927.
- It takes parts or shapes as primitive, instead of points.
- The first axioms of a mereological/mereotopological system are often:

**Part:**  $y = \text{pt}(x)$

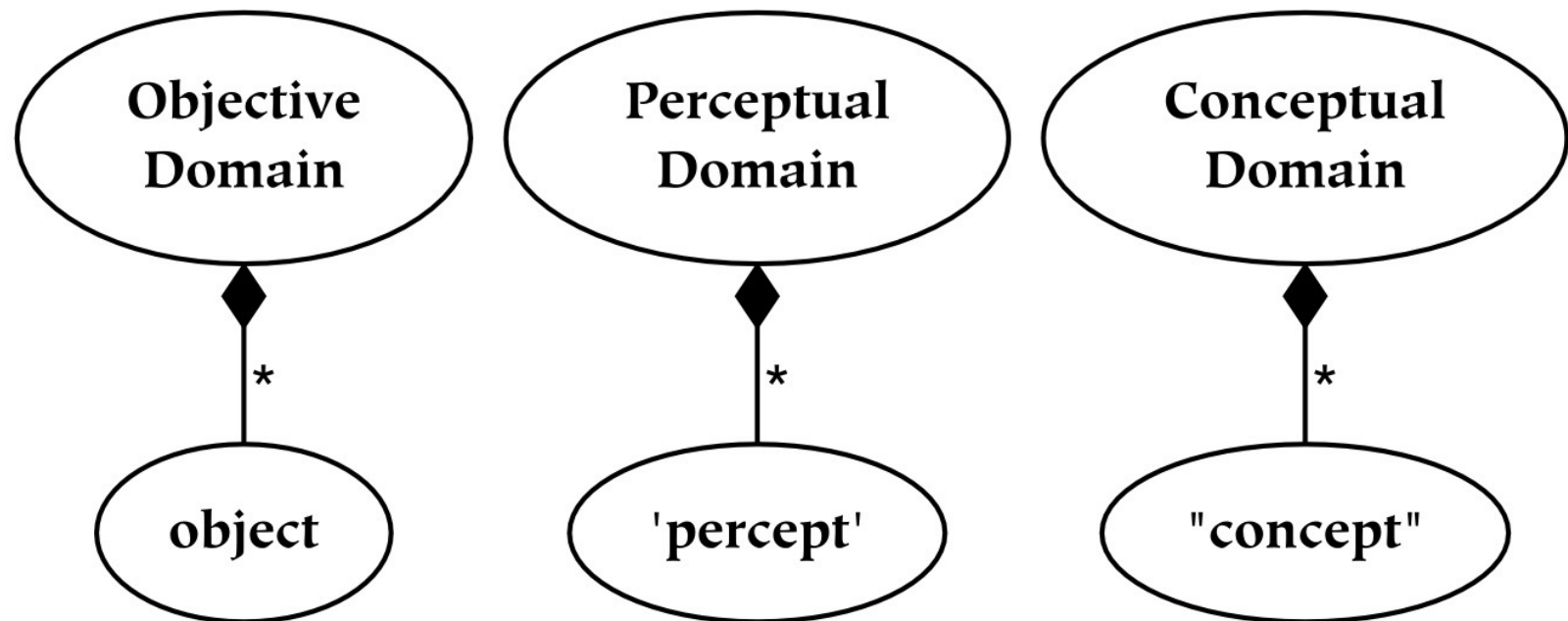
**Proper part:**  $y = \text{ppt}(x) \rightarrow y = \text{pt}(x) \wedge y \neq x$

**Overlap:**  $O(x, y) \rightarrow \exists(z) : z = \text{pt}(x) \wedge z = \text{pt}(y)$

# CST : THE DOMAIN(S)

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- In Cognitive Set Theory, physical reality and mental reality are represented as N-dimensional spaces. Mental space is further divided into perceptual (sensory) space and conceptual (abstract) space.



# CST : RELATIONS

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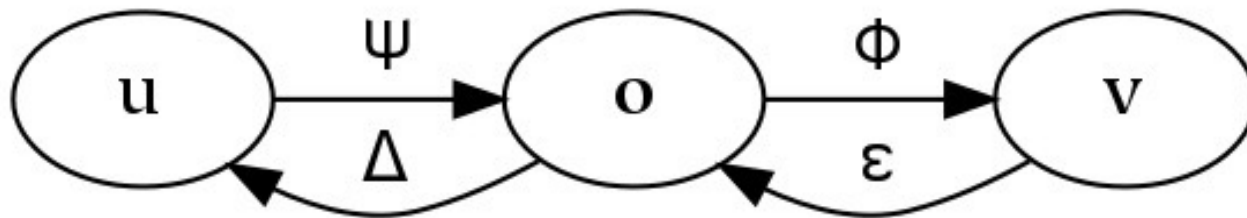
- A common psychological model of how these spaces interact is that mental space is a referential map of physical space. For example, the concept "bicycle" refers to the subjective percept 'bicycle', which in turn refers to the object in the world known as a bicycle.
- Each of these things exists within a similar isomorphic context within its own domain: just as the perception of a bike contains two perceptions of wheels, the concept of a bike contains two wheel-concepts.



# CST : MODEL

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- If we assume that all information that flows from physical reality (U) to conceptual reality (V) passes through a perceptual stage (O), we require four relations:



perception ( $\Psi$ ):	Objective $\rightarrow$ Perceptual
communication ( $\Delta$ ):	Perceptual $\rightarrow$ Objective
conceptualization ( $\Phi$ ):	Perceptual $\rightarrow$ Conceptual
naming ( $\epsilon$ ):	Conceptual $\rightarrow$ Perceptual

# CST : PERCEPTS : POINT-FREE SPACE

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- To reiterate, topology generally uses set theory as its mathematical foundation, with points as the elements of those sets, but defining shapes in terms of sets of points is not psychologically valid.
- Mentally, although we may divide our consciousness into many small parts, there is no reason to assume that our psychological reality *begins* with points, and a number of reasons not to do so.
- Therefore, the (set-theoretic) elements of the CST spatial model are not points: they are parts or percepts formed by the division of a larger space or whole.

# CST : PERCEPTS : THE FIRST SHAPES

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- The replacement of points with parts is motivated in part by developmental psychology, according to which the first internal representations are created during the developmental stage called object permanence (Piaget, 1963).
- A good candidate for the first internal representation from a biological point of view is the bodily sense of self, because bodily inputs change much less than external inputs.
- Perceptual part may be divided, but subparts created in this way remain in their respective contextual space: they are not references (i.e. they are sub-symbolic).



# CST : PERCEPTS : HOLISM

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- Mereotopology avoids a number of several counter-intuitive notions associated with set theory such as infinity, infinitesimal quantities, and the difference between open and closed surfaces.
- The mereotopological process is holistic and top-down, as opposed to the reductionistic and bottom-up process of point-set topology.
- Perception is a process of successive division of a larger whole, which plays a counterpart to the successive combination that occurs in set theory.

# CST : CONCEPTS : THE ELEMENTS

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- To reiterate, mereotopology is the basis for the formation of percepts, and set theory is the basis for concepts.
- Ignoring the mereological side of things, we have a version of set theory with perceptual elements where sets correspond to concepts.
- The first sets are simply single-element sets that consist of these perceptual elements. These (first-order) sets are singular (or whole), and may be designated with a symbol.

# CST : CONCEPTS : SETS AND CONCEPTS

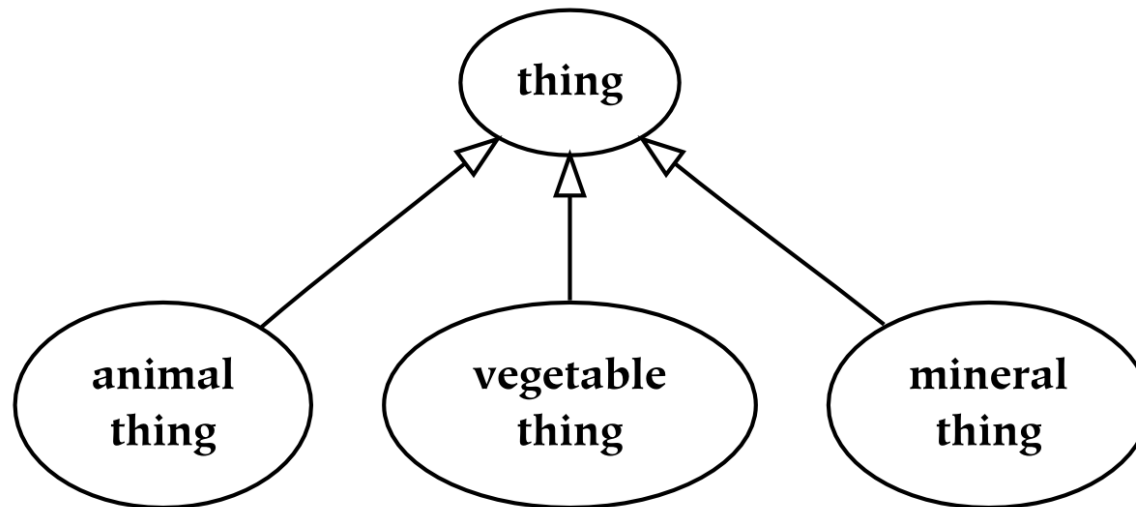
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- The correspondence between mathematical sets and psychological concepts has a number of important features:
  1. Sets are single entities, although they may contain (and compose) a plurality of things.
  2. Sets are typically defined with respect to their contents, independently of other sets.
  3. Sets are in some sense opaque; for example, set membership is not transitive, unlike part membership.

# CST : CONCEPTS : HIGHER-ORDER SETS

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- First-order sets may be recursively collected into higher-order sets.
- Mathematically, the construction of sets out of other sets leads to the Zermelo Hierarchy.
- The depth of the set hierarchy composing a concept corresponds to its psychological level of abstraction.



# CONCLUSION : LINGUA FRANCA

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- Cognitive Set Theory is a simple mental model that uses space as an analogy for the mind, mereology to describe percepts, and set theory to describe concepts.
- The CST Model is not (yet) mathematically rigorous or cognitively complete.
- The combination of formal mathematics and psychology gives it the potential to be a *lingua franca* between multiple disciplines.
- It offers great promise for creating models of cognitive health and the balance between perception and conceptualization.

# THE END

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- Thanks for listening.
- Any feedback is welcome:  
[alec@cognitivesettheory.com](mailto:alec@cognitivesettheory.com)
- Questions?

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Mereology, <https://en.wikipedia.org/wiki/Mereology>

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Set Theory, [https://en.wikipedia.org/wiki/Set\\_theory](https://en.wikipedia.org/wiki/Set_theory)



# CST : FORMAL SUMMARY

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<https://cognitivesettheory.com/html/formalSummary.xhtml>

# PSYCHOLOGY : DUAL PROCESS THEORY

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- Cognitive Set Theory is related to a psychological model called Dual Process Theory.
- According to Dual Process Theory, the mind is composed of a fast, parallel, highly emotional system (System 1) and a slower, serial, less emotional system (System 2).

**System 1:** Independent of working memory, fast, parallel, associative, knows implicitly, relies on basic emotions.

**System 2:** Utilizes working memory, slow, serial, rule-based, knows explicitly, relies on complex emotions.

*Evans and Stanovich, 2013*

# PSYCHOLOGY : DUAL PROCESS THEORY

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- Cognitive Set Theory references these Systems from a subjective point of view: System 1 is experienced perceptually (as percepts) and System 2 is experienced *primarily* conceptually (as concepts or thoughts).
- A primary difference between the Dual-Process division and the division in CST is that System 1 presumably consists of percepts *and* first-order concepts. System 2 consists of only higher-order concepts.

# PSYCHOLOGY : BEHAVIORISM, ASSOCIATIONISM, RFT

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- Cognitive Set Theory can be seen as an extension of Behaviorism:

$CS \rightarrow P_1 \rightarrow CR$

$CS \rightarrow P_2 \rightarrow C \rightarrow P_3 \rightarrow CR$

- The difference between these two models is whether the response is continuous or categorical.
- Associationism entails making bidirectional links between different concepts, C.
- Relational Frame Theory would entail having *relations* instead of associations between different concepts.

# PSYCHOLOGY : GESTALT PSYCHOLOGY

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- Cognitive Set Theory is a holistic theory, so it plays nicely with the concept of *ground* in Gestalt psychology, from which is isolated the figure.
- The following Stimulus  $\rightarrow$  (Percept  $\rightarrow$  Concept)\* loop shows how conception serves to reinforce particular ways of perceiving. Whether it actually alters perception may be framed as “ $P \rightleftharpoons P_n$ ”.

$CS \rightarrow P \rightarrow C_1 \rightarrow P_1 \rightarrow C_1 \rightarrow P_1$

$CS \rightarrow P \rightarrow C_2 \rightarrow P_2 \rightarrow C_2 \rightarrow P_2$

# PSYCHOLOGICAL CATEGORIES : PROTOTYPES AND EXEMPLARS

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- In cognitive science, categories are sometimes assumed to have prototypes: a single member which has the “average” of all characteristics of the examples of that category.
- The exemplar account holds that the elements that compose the category are what constitutes that category.
- In CST, this debate becomes a false dichotomy. Assuming that a category is formed by exposure to individual exemplars, the perceptual memory of the exemplars and the concept they constitute may both be maintained.
- However, if a category is learned not by perceptual exposure but by definition, then there would be no exemplars. This corresponds to the set-theoretic distinction between forming sets by intension (via WFF) and forming sets by extension (via subsets).

# LINGUISTICS : SEMANTICS AND SYNTAX

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- The distinction between semantics and syntax corresponds to the difference between System 1 and System 2.
- The relationship of syntax to set theory is in some sense a proposal of a universal grammar.

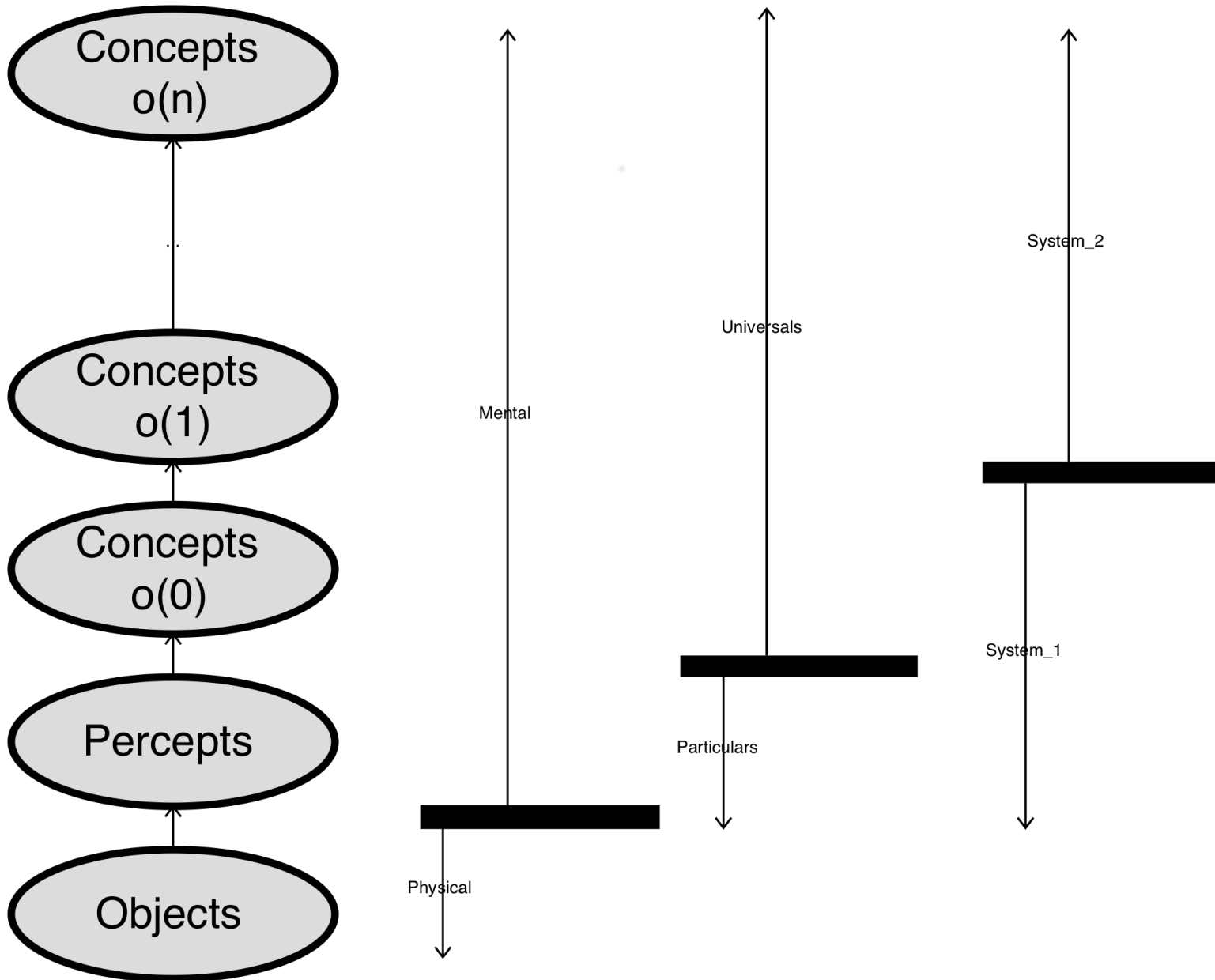
**Merge** (usually capitalized) is one of the basic operations in the [Minimalist Program](#), a leading approach to [generative syntax](#), when two syntactic objects are combined to form a new syntactic unit (a [set](#)). Merge also has the property of [recursion](#) in that it may apply to its own output: the objects combined by Merge are either [lexical items](#) or sets that were themselves formed by Merge. This recursive property of Merge has been claimed to be a fundamental characteristic that distinguishes language from other cognitive faculties. As [Noam Chomsky](#) (1999) puts it, Merge is "an indispensable operation of a recursive system ... which takes two syntactic objects A and B and forms the new object  $G=\{A,B\}$ " (p. 2)

[https://en.wikipedia.org/wiki/Merge\\_\(linguistics\)](https://en.wikipedia.org/wiki/Merge_(linguistics))



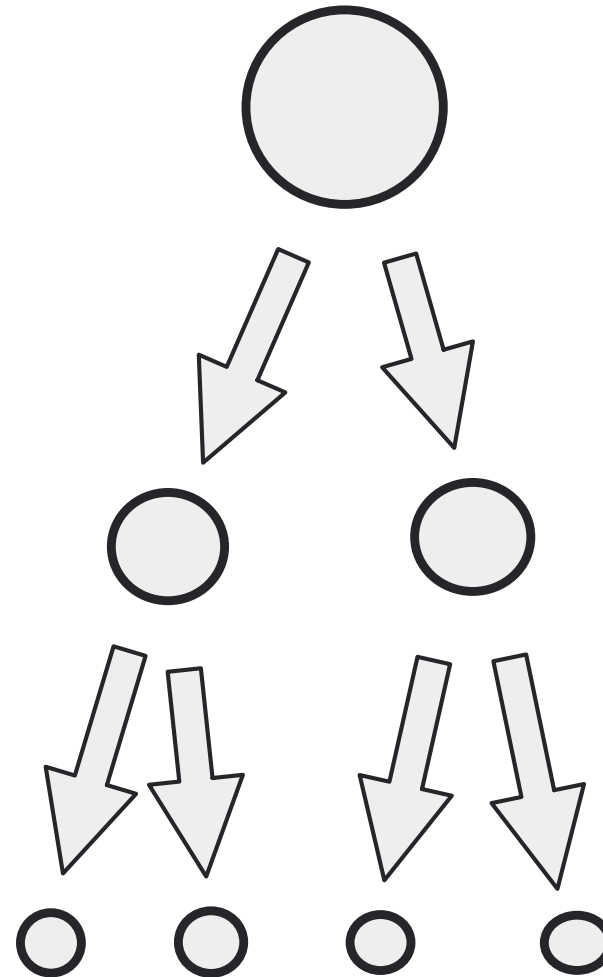
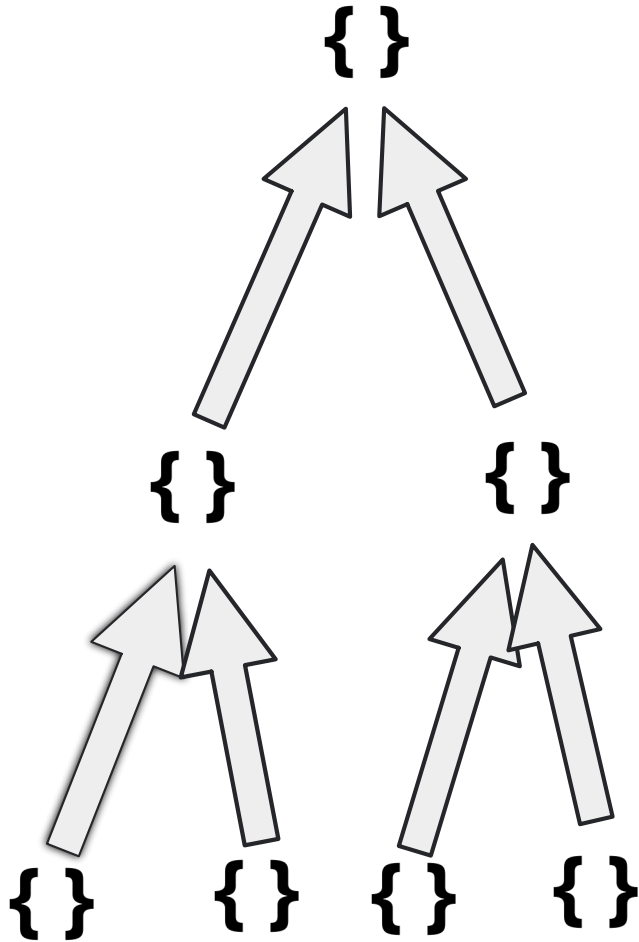
# DIAGRAM : EPISTEMIC LEVELS

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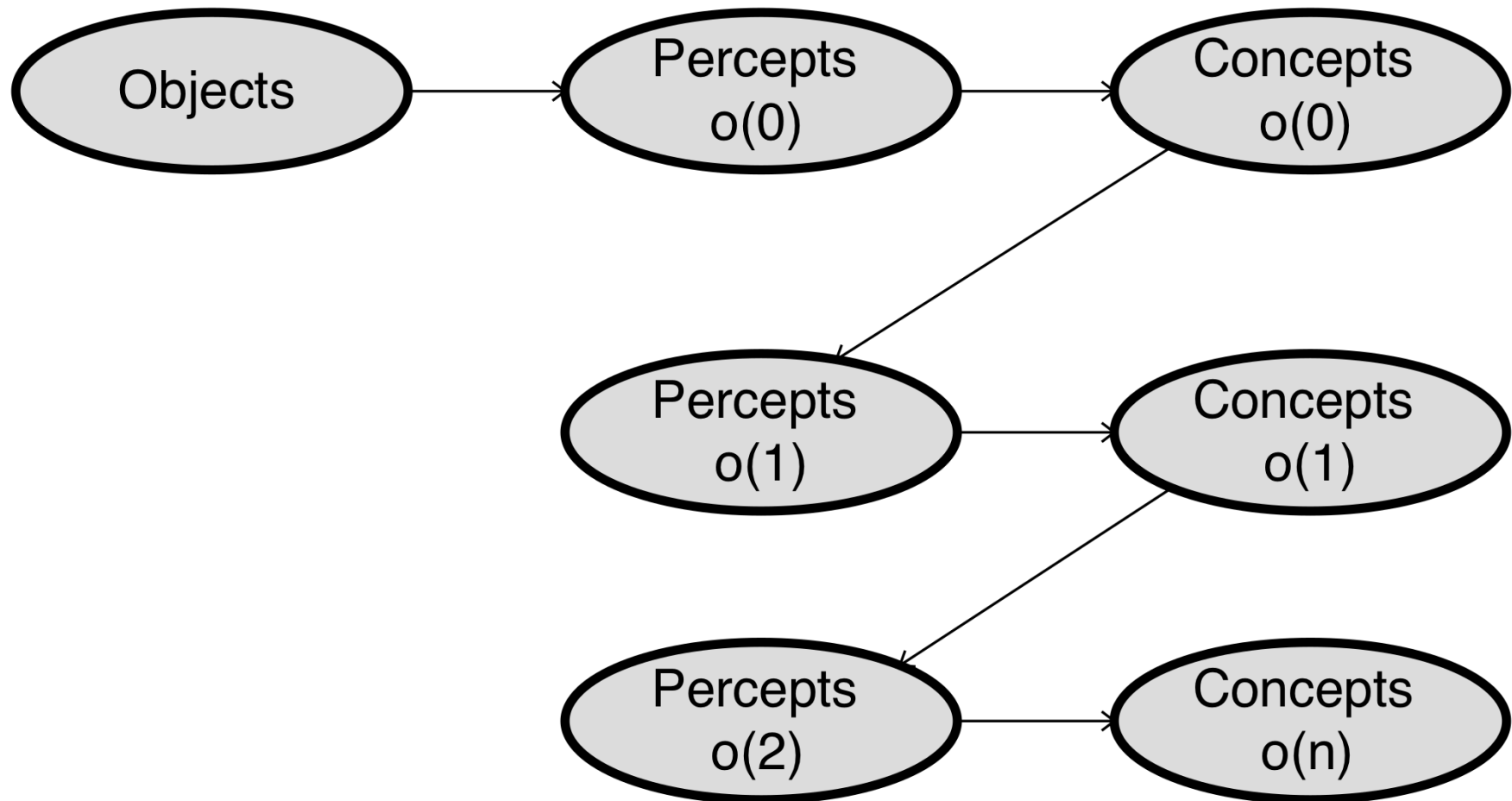
# DIAGRAM : SETS VS PARTS

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# DIAGRAM : RECURSIVE CONCEPTUALIZATION

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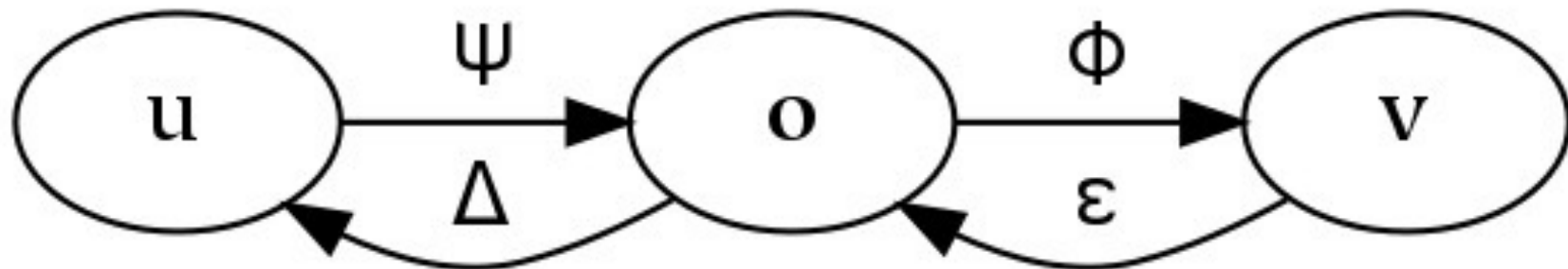
# DIAGRAM : CST MODEL

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Physical (**U**)

Perceptual (**O**)

Conceptual (**V**)



perception ( $\Psi$ ):

Objective  $\rightarrow$  Perceptual

communication ( $\Delta$ ):

Perceptual  $\rightarrow$  Objective

conceptualization ( $\Phi$ ):

Perceptual  $\rightarrow$  Conceptual

naming ( $\epsilon$ ):

Conceptual  $\rightarrow$  Perceptual

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